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The principle of pressure decomposed for flow over a flat plate and its application on the magnetic fluids

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Abstract

According to the diffusion-parabolized Navier-Stokes (DPNS) equations theory and the physical characteristic analysis of the boundary layer of flat plate flow, it is first put foreword that the velocity grade and the pressure grade are different in different layer of the boundary layer and outlet flow field in the tangential direction of the wall surface, which is different from the Prandtl's boundary layer theory using an unitary freestream velocity grade and an unitary pressure grade, and we call it as the trible grades theory. According to the trible grades theory of pressure and the Prandtl's boundary layer theory, the principle of pressure decomposed is obtained. Accordingly, the Navier-Stokes ferrohydrodynamic (FH) equations are simplified, the principle of pressure decomposed of the magnetic fluid is obtained, there are three fluid flows which are controlled by three basic equations: the Euler FH (EFH) equations, diffusion-parabolized FH (DPFH) equations and Navier-Stokes FH (NSFH) equations. The three basic equations have different mathematical characteristics; there is a great disparity in domains of the three basic flows and the domains of EFH equations and DPFH equations are very small. Therefore, adopting EFH-DPFH-NSFH equations system to analyze and compute high Re number magnetic fluid flows over bodies is a logical approach. Our results show that if the discrete grids are not fine enough, numerically simulating the viscous magnetic fluid with the full NSFH equations are equivalent to simulating the magnetic fluid with DPFH equations or with EFH-DPFH-NSFH equations system.

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1. The pressure decomposed principle

In the succeeding section we will give the pressure decomposed principle for the interacting shear flow over a flat plate, which is defined as the flow with convection-dominant in the tangential direction of the wall surface and convection-diffusion competing in the normal direction. That is to say, there exists main-stream direction for an interacting shear flow. Therefore, the coordinate must be elected in reason to ascertain the most simple conservation equations. For example, the material coordinate is elected at a flat plate, and the curve coordinate is elected in the

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conditions of curl wall with x-axis along the wall and y-axis upright the wall. For the uncontrolled shear flow as shear layer at inlet, shoot flow and trail flow, one of the coordinate axes will be consistent with the main-stream direction of the free shear flow.

For the interacting shear flows, there exists a diffuse parabolized layer between the classical Prandtl's boundary layer and the inviscid flow layer. To built the pressure decomposed principle, the four basic assumptions will be built for the interacting shear flows as follows

(A) The flow is predominantly parallel near wall, i.e. $u>>v$.

(B) There exists a diffuse parabolized layer between the Prandtl's boundary layer and the inviscid flow layer.

(C) The scales of physical quantities are different in different layer of the interacting shear flow.

(D) In the inviscid layer, the influnce of viscosity is unimportant.

In order to answer the question of how and which terms are negnected, it is necessary, first to estimate the gradients of any terms with sure formation in different regions. Let δ_1 be the Prandtl's boundary layer thickness, δ_2 be the diffuse parabolized layer thickness. And $\delta = \delta_1 + \delta_2$ is called the boundary layer thickness. Compared with the plate length L, the boundary layer thickness is very small, that is to say, $L >> \delta_2 >> \delta_1$, it is easy to know that $L + \delta_2 >> \delta_2$ and $\delta_1 + \delta_2 \sim \delta_2$. It implies that the axial derivatives of velocity components are much smaller than the transverse derivatives of those same components, the boundary layer thickness is much smaller than the characteristic scale, and the Prandtl's boundary layer thickness is much smaller than the diffuse parabolized layer thickness. In the inviscid flow region the flow is frictionless and potential, and the Euler equations are valid.

The pressure gradients in the tangential direction of the wall surface are kept in the equations thus allowing for wave propagations. The pressure is passive stress which is decided by the momentum and the model of the fluid, and the scales of the pressure are given by two scale exponents, in different regions of flowfield or for different fluid flows, the scales of the pressure are different.

In the classical Prandtl boundary-layer theory, v is of the order δ_1 and Re ~ δ_1^2, then $\partial v/\partial x$ and $\partial^2 v/\partial x^2$ are also of the order δ_1. From the normal momentum equation we may infer that $\partial p/\partial y$ is of the order δ_1 which is very small, then the Prandtl boundary-layer equations which contain the normal momentum equation $\partial p/\partial y=0$ are valid. It is easy to show that $p=p_1(x)$ in the Prandtl boundary-layer. In the outflow region, $p=p(x, y)$ can't be decomposed as total of the downstream and cross-stream direction pressure components.

The question is which form is valid for pressure in the diffusion parabolized layer. According to the decomposed form of pressure and the assumption (B) and (C), we should like to build the principle of pressure decomposed as follows

The principle of pressure decomposed *In the Prandtl boundary layer and the diffusion parabolized layer for flow over a flat plate satisfying four basic assumptions (A)-(D), the total pressure can be decomposed as* $p=p_{1i}(x)+p_{2i}(y)(i=1,2)$, *where* $p_{1i}(x)$ *and* $p_{1i}(x)$ *are the downstream and cross-stream direction pressure components. Further more,* $p_{12}(y)=0$ *and* $p=p_{11}(x)$ *in Prandtl boundary-layer.*

Based on the principle of pressure decomposed, we will analysis the thickness of boundary layer and build the principle of pressure gradient and other corresponding theorys. The principle of pressure decomposed will be proved numerically in this work, and further theoretical and experimental investigations of this problem will be later undertaken and enriched in other works

2. The pressure decomposed principle of the magnetic fluids

For the interacting shear flows of the magnetic fluids, there exists a magnetic layer between the classical Prandtl's boundary layer and the wall. To built the pressure decomposed principle, the four basic assumptions will be built for the interacting shear flows as follows

(A) The flow is predominantly parallel near wall, i.e. $u>>v$.

(B) There exists a diffuse parabolized layer between the Prandtl's boundary layer and the inviscid flow layer.

(C) The scales of physical quantities are different in different layer of the interacting shear flow.

(D) In the inviscid layer, the influnce of viscosity is unimportant.

(E) The thickness of magnetic layer is decided by magnetic fluids which is less effected by velocity and temperature of fluid. Then the thickness of magnetic layer is constant.

Let d be the thickness of magnetic layer, we assume that $d \sim \delta_2$. The question is which form is valid for pressure in the diffusion parabolized layer. According to the decomposed form of pressure and the assumption (B) and (C), we should like to build the principle of pressure decomposed as follows

The principle of pressure decomposed of the magnetic fluids *For the interacting shear flows of the magnetic fluids, there exists a magnetic layer between the classical Prandtl's boundary layer and the wall. In the Prandtl boundary layer and the diffusion parabolized layer for flow over a flat plate satisfying four basic assumptions (A)-(D), the total pressure can be decomposed as* $p=p_{1i}(x)+p_{2i}(y)(i=1,2)$, *where* $p_{1i}(x)$ *and* $p_{1i}(x)$ *are the downstream and cross-stream direction pressure components. Further more,* $p_{12}(y)=0$ *and* $p=p_{11}(x)$ *in Prandtl boundary-layer.*

3. The diffusion-parabolized FH equations

By the principle of pressure decomposed, we obtain the following diffusion parabolized Navier-Stokes(DPNS) equations:

$$\partial u/\partial x + \partial v/\partial y = 0, \tag{3.1}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \eta\frac{\partial^2 u}{\partial y^2}, \tag{3.2}$$

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \eta\frac{\partial^2 v}{\partial y^2} \tag{3.3}$$

By the principle of pressure decomposed of the magnetic fluids, we obtain the following diffusion parabolized ferrohydrodynamic (DPFH) equations:

$$\partial u/\partial x + \partial v/\partial y = 0 \tag{3.4}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p^*}{\partial x} + \eta\frac{\partial^2 u}{\partial y^2} + \mu_0\left(M_1\frac{\partial H_1}{\partial x} + M_2\frac{\partial H_1}{\partial y}\right) \tag{3.5}$$

$$\rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial p^*}{\partial y} + \eta\frac{\partial^2 v}{\partial y^2} + \mu_0\left(M_1\frac{\partial H_2}{\partial x} + M_2\frac{\partial H_2}{\partial y}\right) \tag{3.6}$$

Where $p^*=p^0+\rho_0 H^2/2$ (see [11]).

4. Conclusion

The velocity grade and the pressure grade are put foreword in different layer of the boundary layer and outlet flow field in the tangential direction of the wall surface. The principle of pressure decomposed and the principle of pressure decomposed of the magnetic fluids are obtained.

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Reference

[1] Gao Z., The simplified Navier-Stokes equations and simultaneous solution of externally inviscid flow and viscous boundary layer flow, Acta Mechanica Sinica (in china), 14(6)(1982)606-661 (see: Z. Gao, Project of simultaneous solution of externally inviscid flow and viscous boundary layer flow, and simultaneous solution of externally inviscid caudal flow and viscous caudal boundary layer flow, Science Bulletin in Mechanics Institute of CAS, 1967).
[2] Gao Z., The simplified Navier-Stokes, Science in China (series A) , 17(10)(1987) 1058-1070.
[3] Gao Z., Viscous-inviscid interaction flow theory, Acta Mechanica Sinica (in china), 22(1)(1990) 9-19.
[4] Yong Yu, A new CFD validation method with application to verify computations of near wall flows, 39th AIAA fluid dynamics conference, 22-25, June 2009, San Antonio, nTexas.
[5] Davis R.T., Numerical solution of the hypersonic viscous shock-layer-equations, AIAA J, 1970, 33, 843-851.
[6] Rudman S., Rubin S G, Hypersonic viscous flow over slender bodies with sharp leading edges, AIAA Journal, 10(1968)1883-1890.
[7] Madavan N. K., Pletcher R. H., Prediction of laminar separated flows using the partially parabolized Navier-Stokes equations. Computational Fluid Dynamics Conference, 6th, Danvers, MA, July 13-15, 1983, Collection of Technical Papers (A83-39351 18-02). New York, American Institute of Aeronautics and Astronautics, 1983, p. 463-472.
[8] Tannehill T.C., Anderson D.A., Pletcher R.H., Computational Fluid Mechanics and Heat Transfer (Second edition), New York, Taylor & Francis, 1997.
[9] George W. Sutton and Arthur Sherman, Engineering Magnetohydrodynamics, McGRAW-HILL BOOK COMPANY, New York, 1965.
[10] H. Yamaguchi, Engineering Fluid mechanics, Springer-Verlag, New York, 2009.
[11] H. Yamaguchi, Z. Zhang, S. Shuchi, K.Shimad, Heat transfer characteristics of magnetic fluid in a partitioned rectangular box, Journal of Magnetism and Magnetic Materials 252(2002)203.
[12] H. Yamaguchi, X.R. Zhang, S. Higashi, M. Li, Study on power generation using electro-conductive polymer and its mixture with magnetic fluid, Journal of Magnetism and Magnetic Materials 320 (2008) 1406–1411.